



# Why Functional Programming Matters

John Hughes  
Mary Sheeran



# Functional Programming à la 1940s

- Minimalist: who needs booleans?
- A boolean just *makes a choice!*

**true**    **x y = x**

**false** **x y = y**

- We can *define* if-then-else!

**ifte** **bool t e =**  
**bool t e**

# Who needs integers?

- A (positive) integer just *counts loop iterations!*

**two** **f** **x** = **f** (**f** **x**)

**one** **f** **x** = **f** **x**

**zero** **f** **x** = **x**

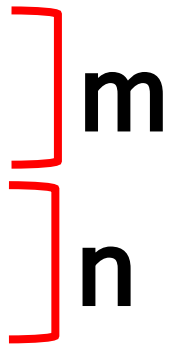
- To recover a "normal" integer...

```
*Church> two (+1) 0  
2
```

# Look, Ma, we can add!

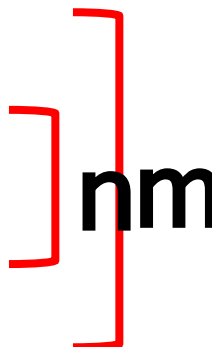
- Addition by *sequencing* loops

$$\mathbf{add\ m\ n\ f\ x = m\ f\ (n\ f\ x)}$$



- Multiplication by *nesting* loops!

$$\mathbf{mul\ m\ n\ f\ x = m\ (n\ f)\ x}$$



```
*Church> add one (mul two two) (+1) 0
5
```

# Factorial à la 1940

```
fact n =  
  ifte (iszero n)  
    one  
    (mul n (fact (decr n)))
```

```
*Church> fact (add one (mul two two)) (+1) 0  
120
```

# A couple more auxiliaries

- Testing for zero

```
iszero n =  
  n (\_ -> false) true
```

- Decrementing...

```
decr n =  
  n (\m f x-> f (m incr zero))  
  zero  
  (\x->x)  
  zero
```

Booleans, integers, (and other data structures) *can be entirely replaced by functions!*

*"Church encodings"*

Early versions of the Glasgow Haskell compiler actually implemented data-structures this way!



*Alonzo Church*



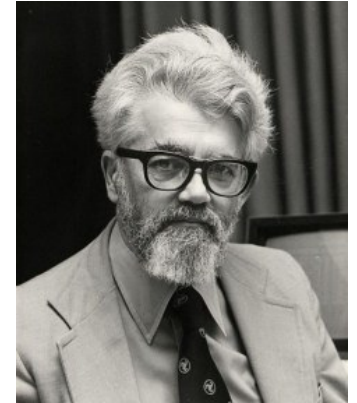




The type-checker needs a *little bit* of help

```
fact ::  
  (forall a. (a->a) ->a->a) ->  
  (a->a) -> a -> a
```

# Factorial à la 1960



```
(LABEL FACT (LAMBDA (N)
  (COND ((ZEROP N) 1)
        (T (TIMES N (FACT (SUB1 N)))))))
```

## Higher-order functions!

```
(MAPLIST FACT (QUOTE (1 2 3 4 5)))

(1 2 6 24 120)
```

# The Next 700 Programming Languages

P. J. Landin

*Univac Division of Sperry Rand Corp., New York, New York*

“... today ... 1,700 special programming languages used to ‘communicate’ in over 700 application areas.”—*Computer Software Issues*, an American Mathematical Association Prospectus, July 1965.



## Factorial in ISWIM

`fac(5)`

where `rec fac(n) =`

`(n=1) → 1;`

`n*fac(n-1)`

# Laws

(MAPLIST F (REVERSE L))  $\equiv$  (REVERSE (MAPLIST F L))

What's the point of two different ways to do the same thing?

Wouldn't *two* facilities be better than one?

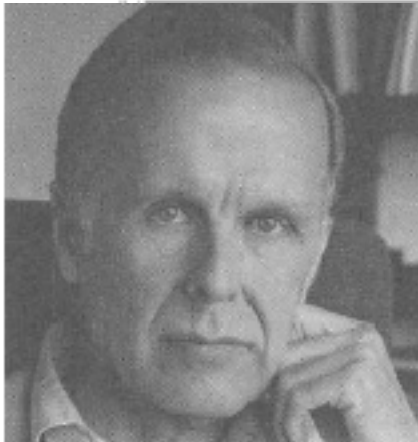
**Expressive power should be by design, rather than by accident!**



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# Can Programming Be Liberated from the von Neumann Style? A Functional Style and Its Algebra of Programs

John Backus  
IBM Research Laboratory, San Jose



Turing award 1977

[Paper 1978](#)

**Conventional programming  
languages are growing ever  
more enormous,  
but not stronger.**



Inherent defects at the most basic level cause them to be both **fat** and **weak**:

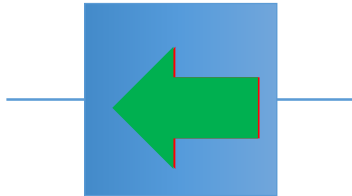
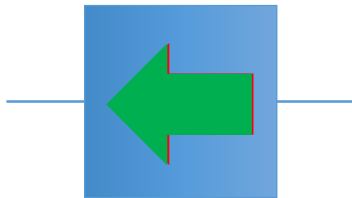
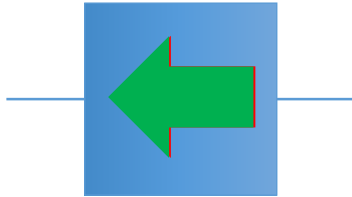
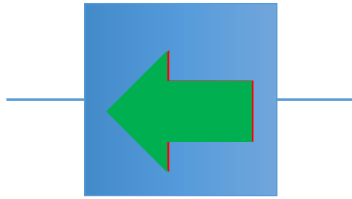
Word-at-a-time



their inability to effectively use  
powerful combining forms  
for building new programs from  
existing ones



apply to all



$\alpha f$

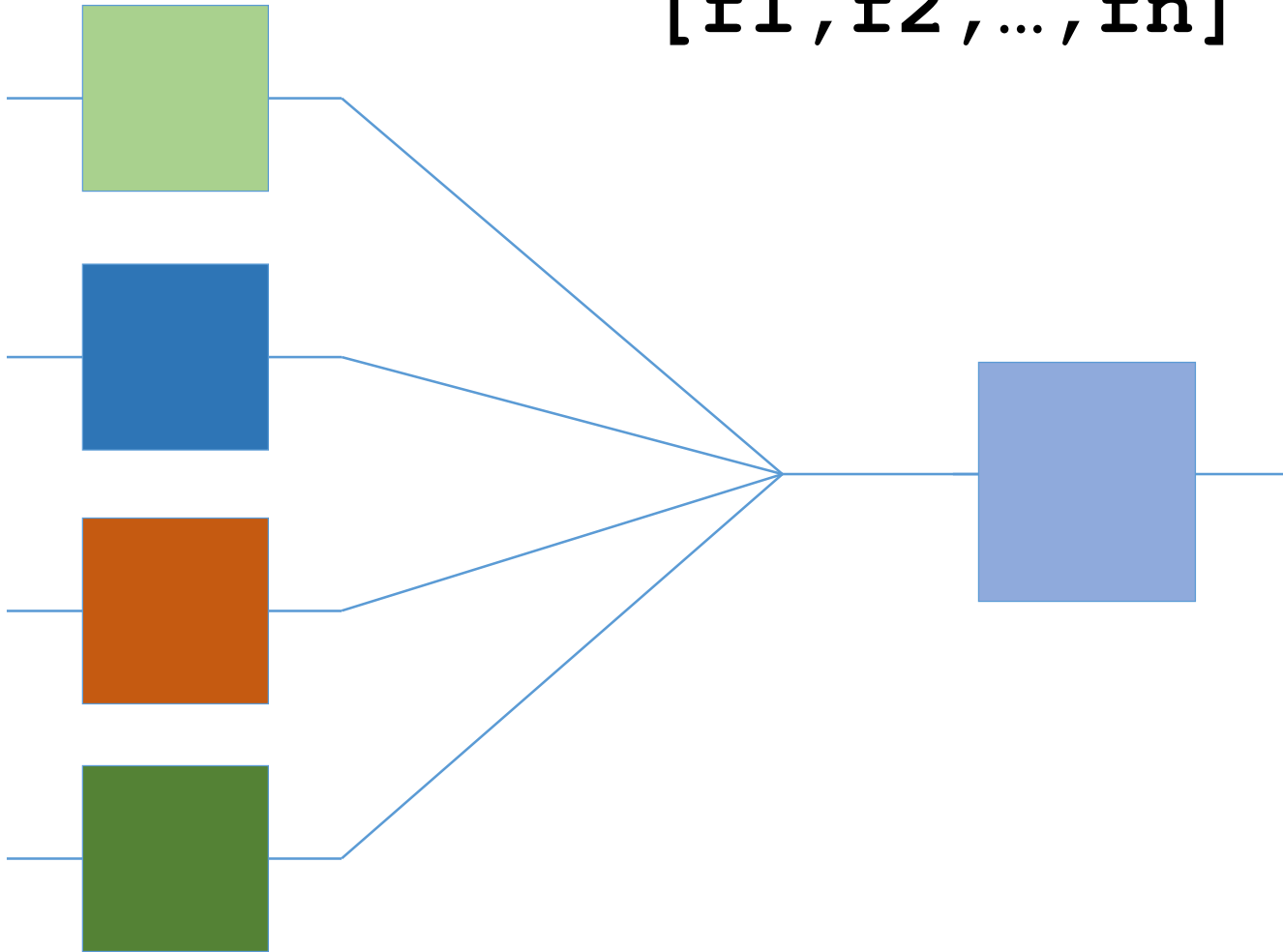
construction



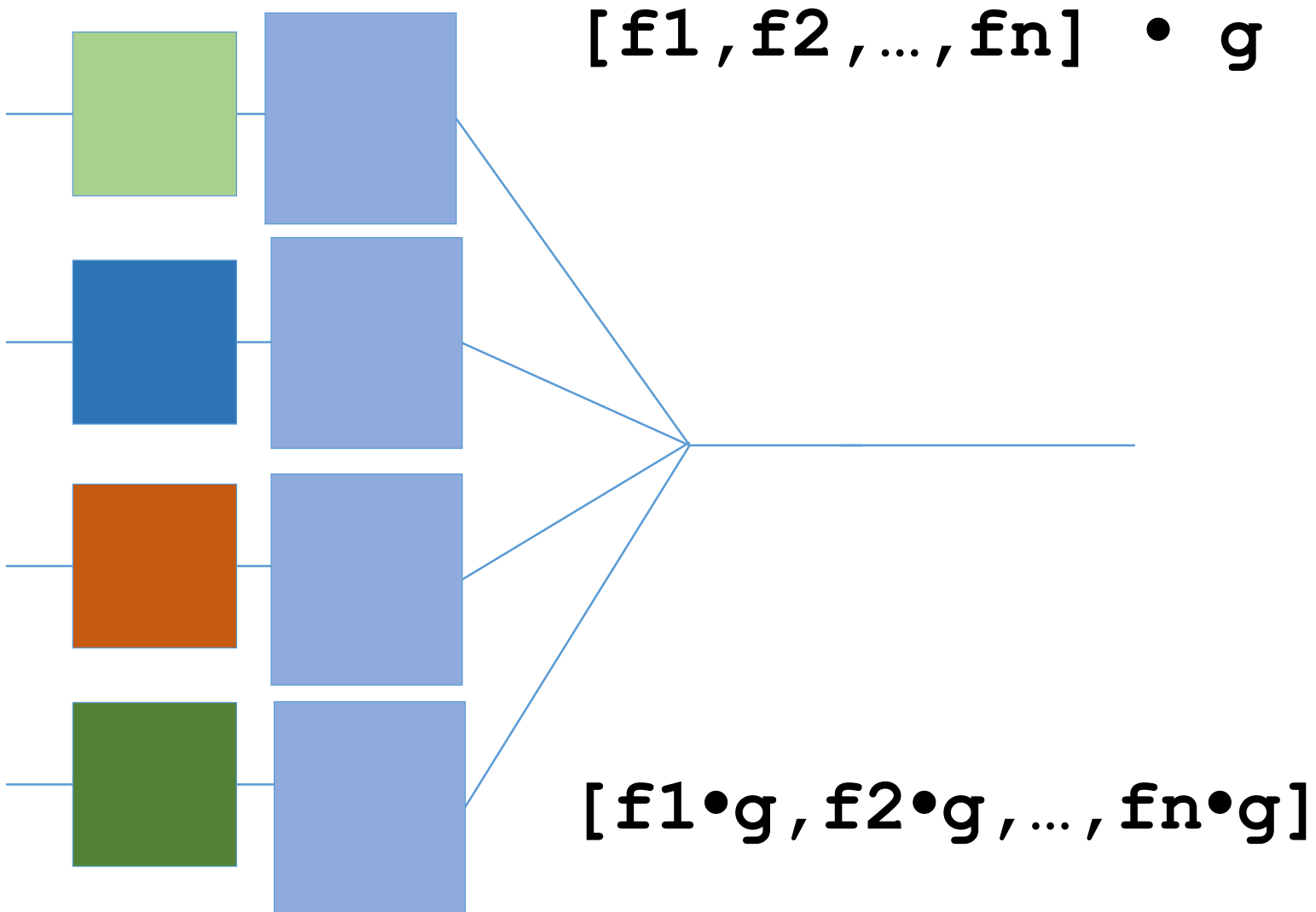
$[f1, f2, f3, f4]$

**their lack of useful  
mathematical properties for  
reasoning about programs**

$[f_1, f_2, \dots, f_n] \cdot g$







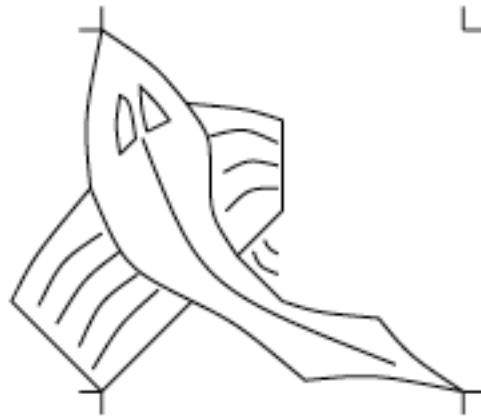
```
c := 0;  
for i := 1 step 1 until n do  
    c := c + a[i] × b[i]
```

```
Def ScalarProduct =  
  (Insert +) • (ApplyToAll ×) • Transpose
```

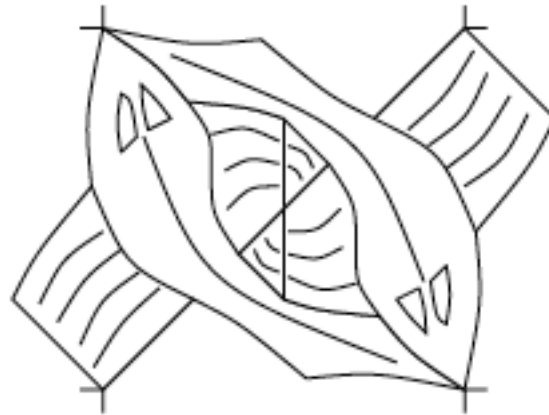
**Def SP = (/ +) • (α ×) • Trans**

*Peter Henderson, Functional Geometry, 1982*

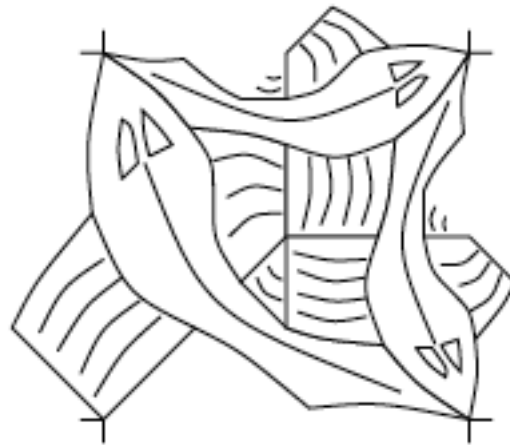




**fish**



`over (fish, rot (rot (fish)))`

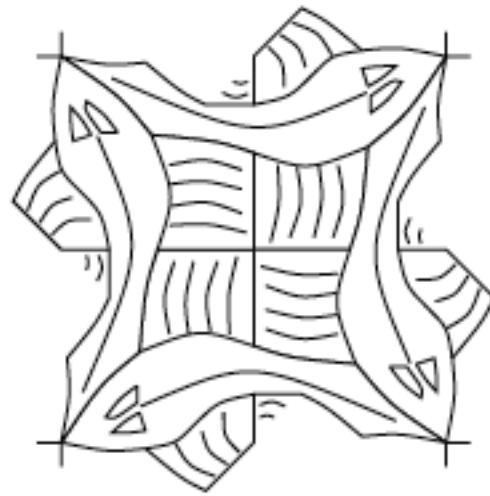


```
t = over (fish, over (fish2, fish3))
```

```
fish2 = flip (rot45 fish)
```

```
fish3 = rot (rot (rot (fish2)))
```

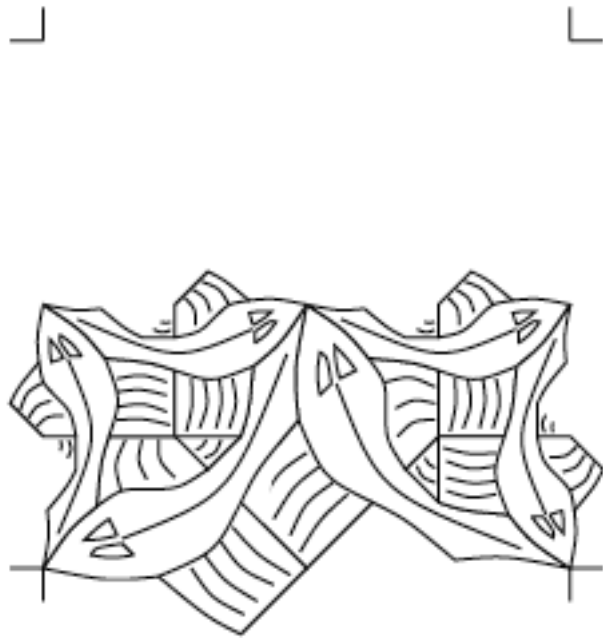




```
u = over (over (fish2, rot (fish2)),  
          over (rot (rot (fish2)),  
                rot (rot (rot (fish2))))))
```

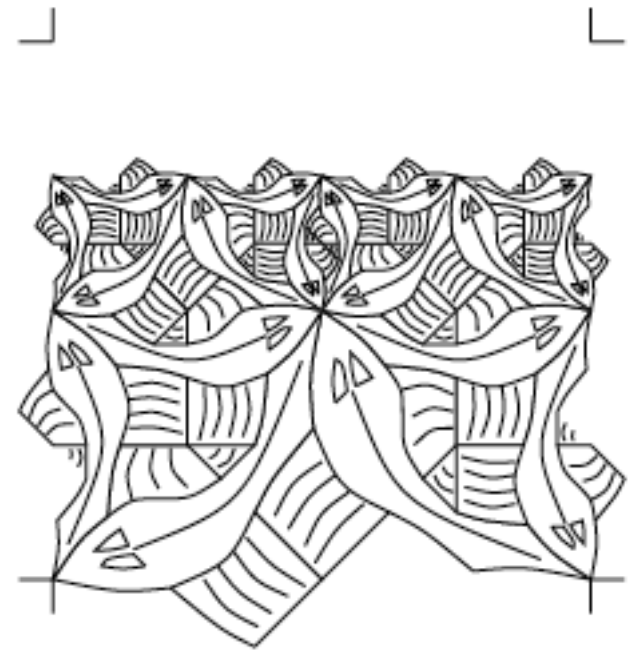
P	Q
R	S

quartet



`quartet(nil, nil,  
rot(t), t)`

`side1`

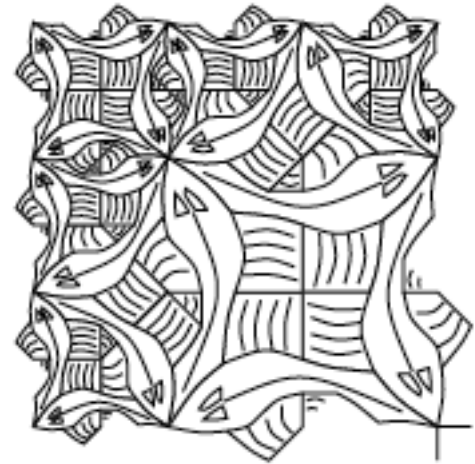


`quartet(side1, side1,  
rot(t), t )`



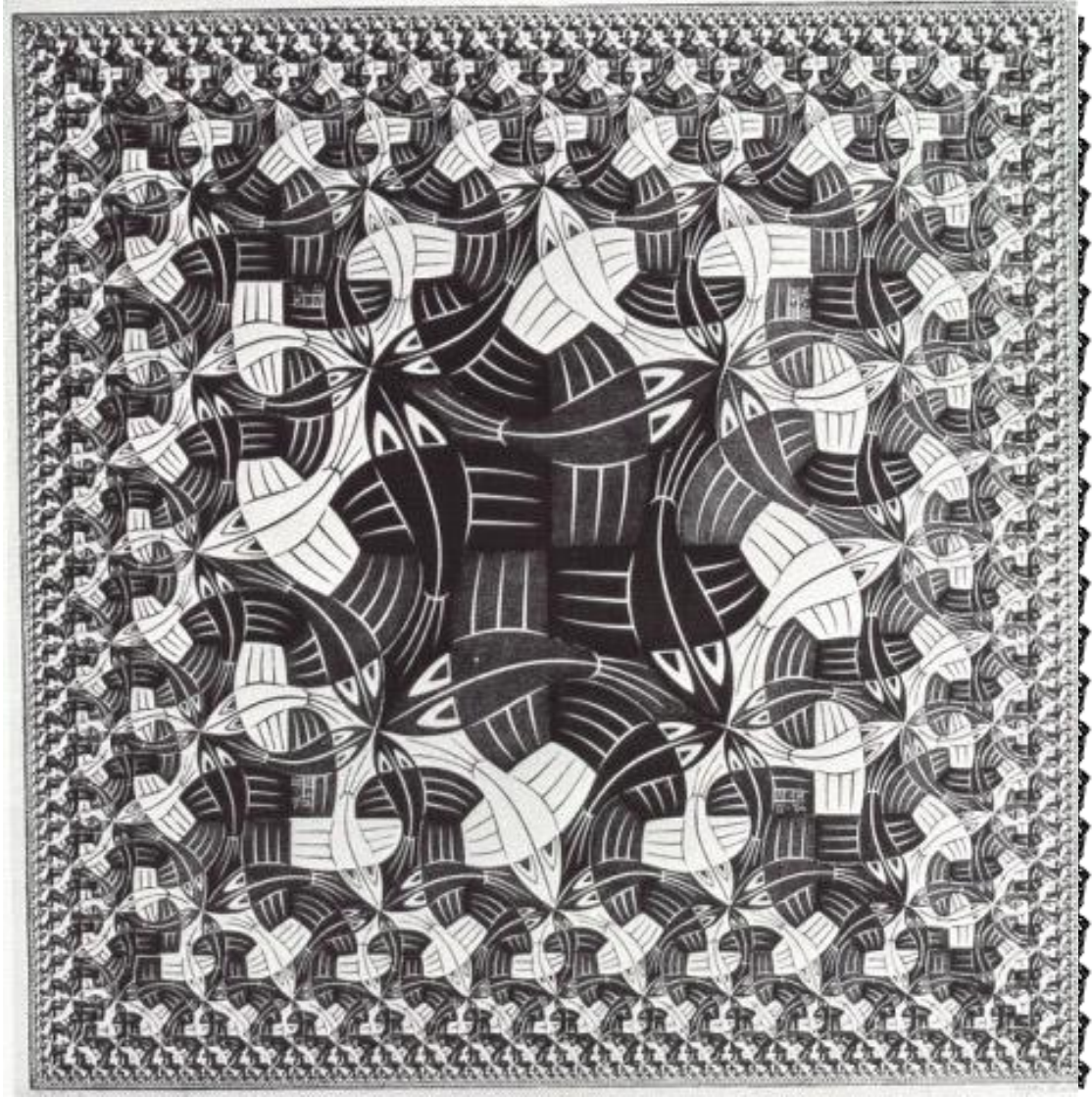
`quartet (nil,nil,nil,u)`

`corner1`



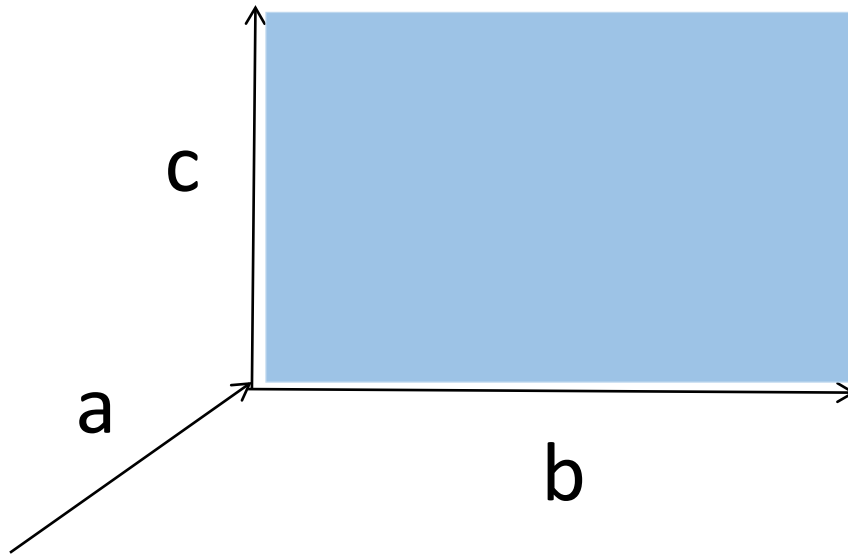
`quartet (corner1,  
side1,  
rot (side1) ,  
u)`

```
squarelimit = nonet(  
    corner,      side,      rot(rot(rot(corner))),  
    rot(side),  u,          rot(rot(rot(side))),  
    rot(corner), rot(rot(side)), rot(rot(corner)))
```



picture = function

picture = function

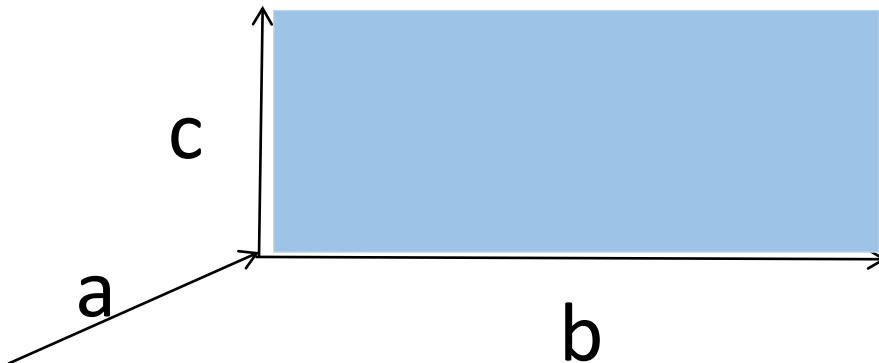




$$\text{over } (p, q) \ (a, b, c) = \\ p(a, b, c) \cup q(a, b, c)$$

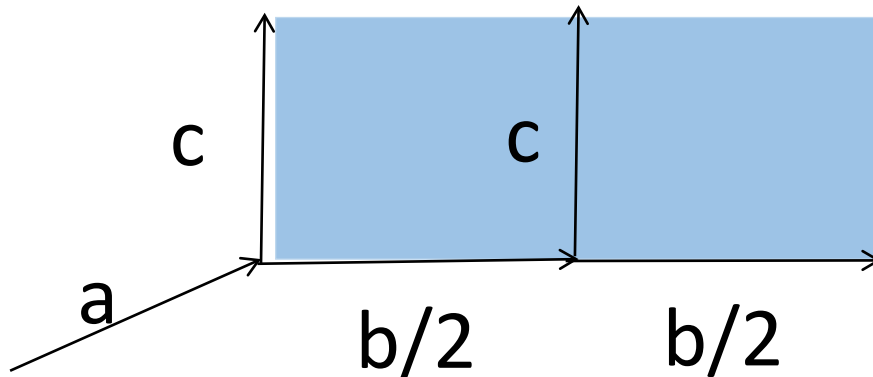
over  $(p, q) (a, b, c) =$   
 $p(a, b, c) \cup q(a, b, c)$

beside  $(p, q) (a, b, c) =$   
 $p(a, b/2, c) \cup q(a+b/2, b/2, c)$

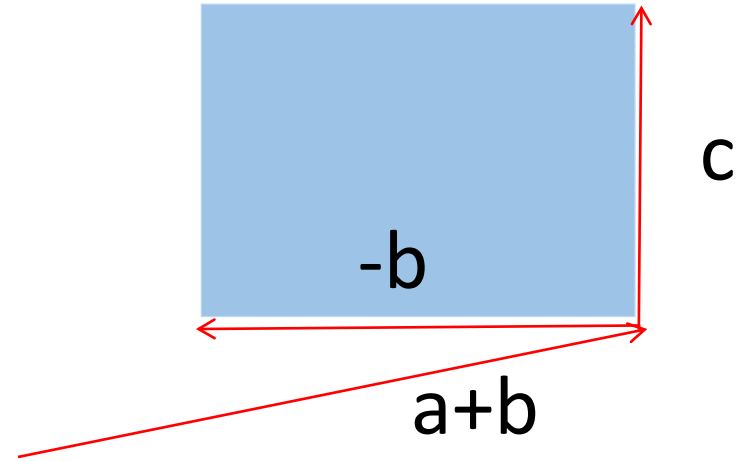
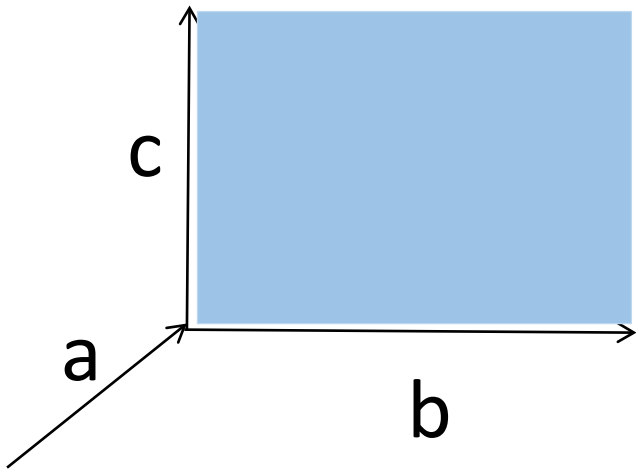


over  $(p, q)$   $(a, b, c) =$   
 $p(a, b, c) \cup q(a, b, c)$

beside  $(p, q)$   $(a, b, c) =$   
 $p(a, b/2, c) \cup q(a+b/2, b/2, c)$



$$\text{rot}(p) \ (a, b, c) = p \ (a+b, c, -b)$$



# Laws

$$\text{rot (above (p , q) )} \\ = \\ \text{beside (rot (p) , rot (q) )}$$



It seems there is a positive correlation between the simplicity of the rules and the quality of the algebra as a description tool.

Whole values

Combining forms

Algebra as litmus test

Whole values

Combining forms

Algebra as litmus test

functions as  
representations

# Haskell vs. Ada vs. C++ vs. Awk vs. ... An Experiment in Software Prototyping Productivity\*

Paul Hudak  
Mark P. Jones

Yale University  
Department of Computer Science  
New Haven, CT 06518  
{hudak-paul, jones-mark}@cs.yale.edu



July 4, 1994





Time 40.0:

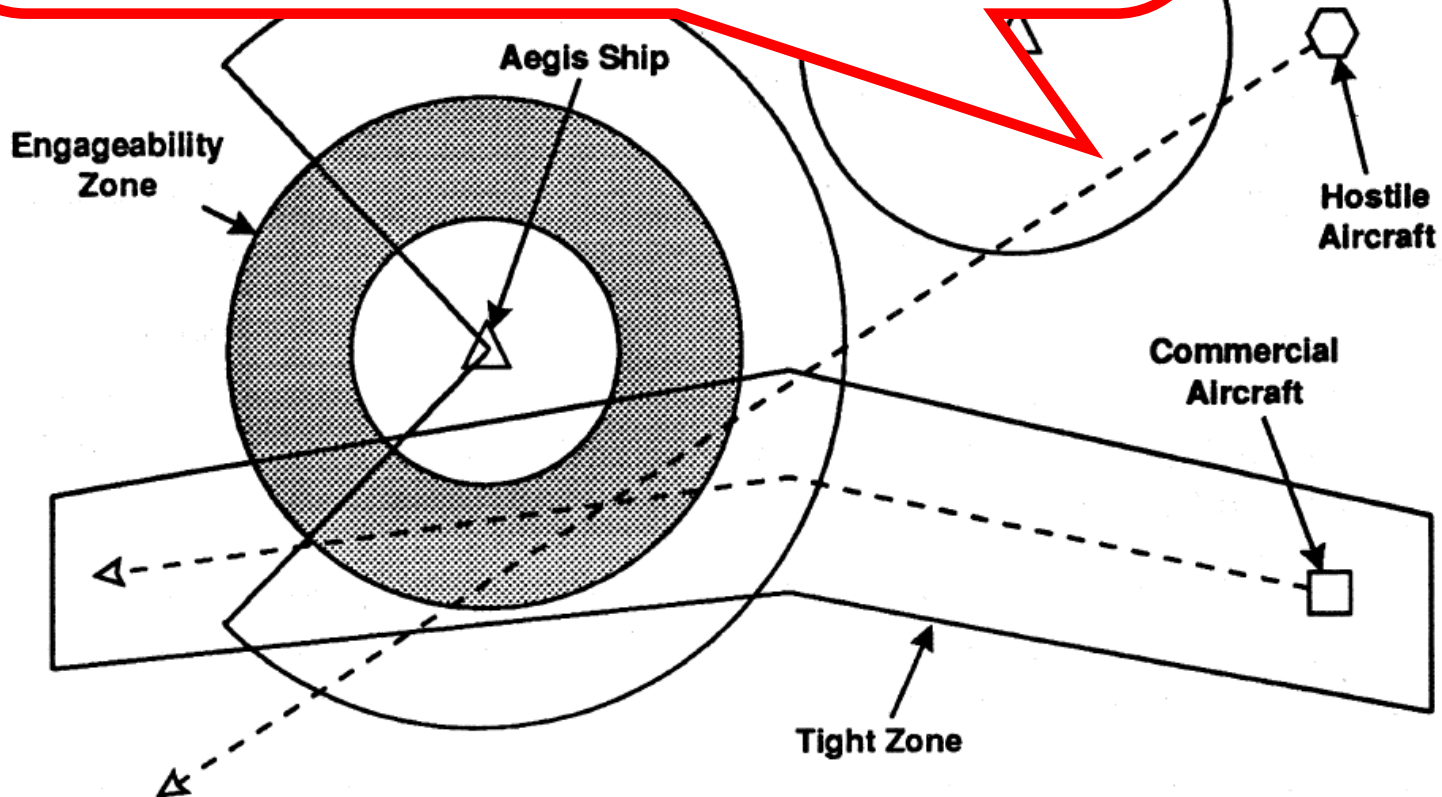
commercial aircraft: (100.0,43.0)

-- In engageability zone

-- In tight zone

hostile craft: (210.0,136.0)

-- In carrier slave doctrine



# Functions as Data

```
> type Region    = Point -> Bool

> circle        :: Radius -> Region
> outside       :: Region -> Region
> (/\)          :: Region -> Region -> Region

> annulus       :: Radius -> Radius -> Region
> annulus r1 r2 = outside (circle r1) /\ circle r2
```

Including 29 lines of inferable type signatures/synonyms

A student, given 8 days to learn Haskell, w/o knowledge of Yale group

Language	Lines of code	Lines of document	Development time (hours)
(1) Haskell	85	465	10
(2) Ada	767	7	23
(3) Ada9X	800	7	28
(4) C++	1105	130	-
(5) Awk/Nawk	250	150	-
(6) Rapide	157	0	54
(7) Griffin	251	0	34
(8) Proteus	293	79	26
(9) Relational Lisp	274	12	3
(10) Haskell	156	112	8

Figure 3: Summary of Prototype Software Development Metrics

# Reaction...

”too cute for its own good”

...higher-order functions just a trick, probably not useful in other contexts

# Lazy Evaluation (1976)



Henderson and Morris  
*A lazy evaluator*



Friedman and Wise  
*CONS should not evaluate  
its arguments*

# “The Whole Value” can be $\infty$ !

- The *infinite list* of natural numbers  
[0, 1, 2, 3 ...]

*Consumer* decides  
how many to  
compute

- All the iterations of a function

`iterate f x = [x, f x, f (f x), ...]`

- A consumer for numerical methods

`limit eps xs =`

*<first element of **xs** within **eps** of its predecessor>*

# Some numerical algorithms

- Newton-Raphson square root

```
sqrt a = limit eps (iterate next 1.0)  
  where next x = (x + a/x) / 2
```

- Derivatives

```
deriv f x =  
  limit eps (map slope (iterate (/2) 1.0))  
  where slope h = (f (x+h) - f x) / h
```

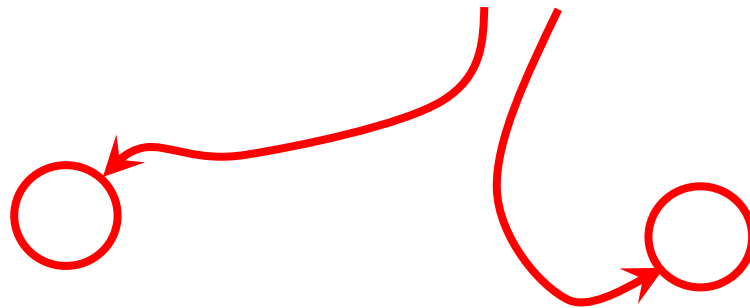
[1, 1/2, 1/4, 1/8...]

*Same convergence check*

*Different approximation sequences*

# Speeding up convergence

The smaller  $h$  is, the better the approximation



Differentiation

Integration

The right answer

$$A + B * h^n$$

An error term



# Eliminating the error term

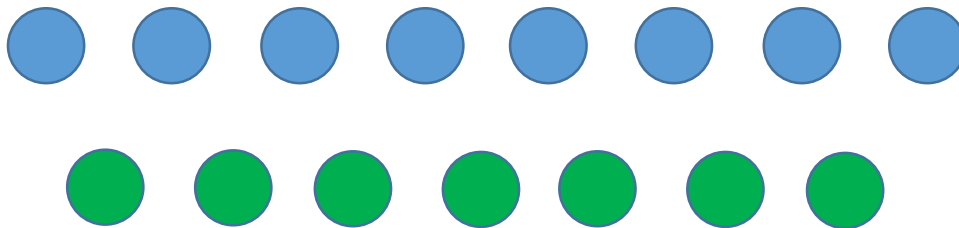
- Given:

$$A + B * h^n$$

$$A + B * (h/2)^n$$

Two successive approximations

- Solve for A and B!



**improve**  $n$  **xs** *converges faster than* **xs**

# Really fast derivative

```
deriv f x =  
  limit eps  
    (improve 2  
      (improve 1  
        (map slope (iterate (/2) 1.0))))
```

*The convergence check*

*The improvements*

*The approximations*

Everything is programmed *separately* and easy to understand—thanks to “whole value programming”

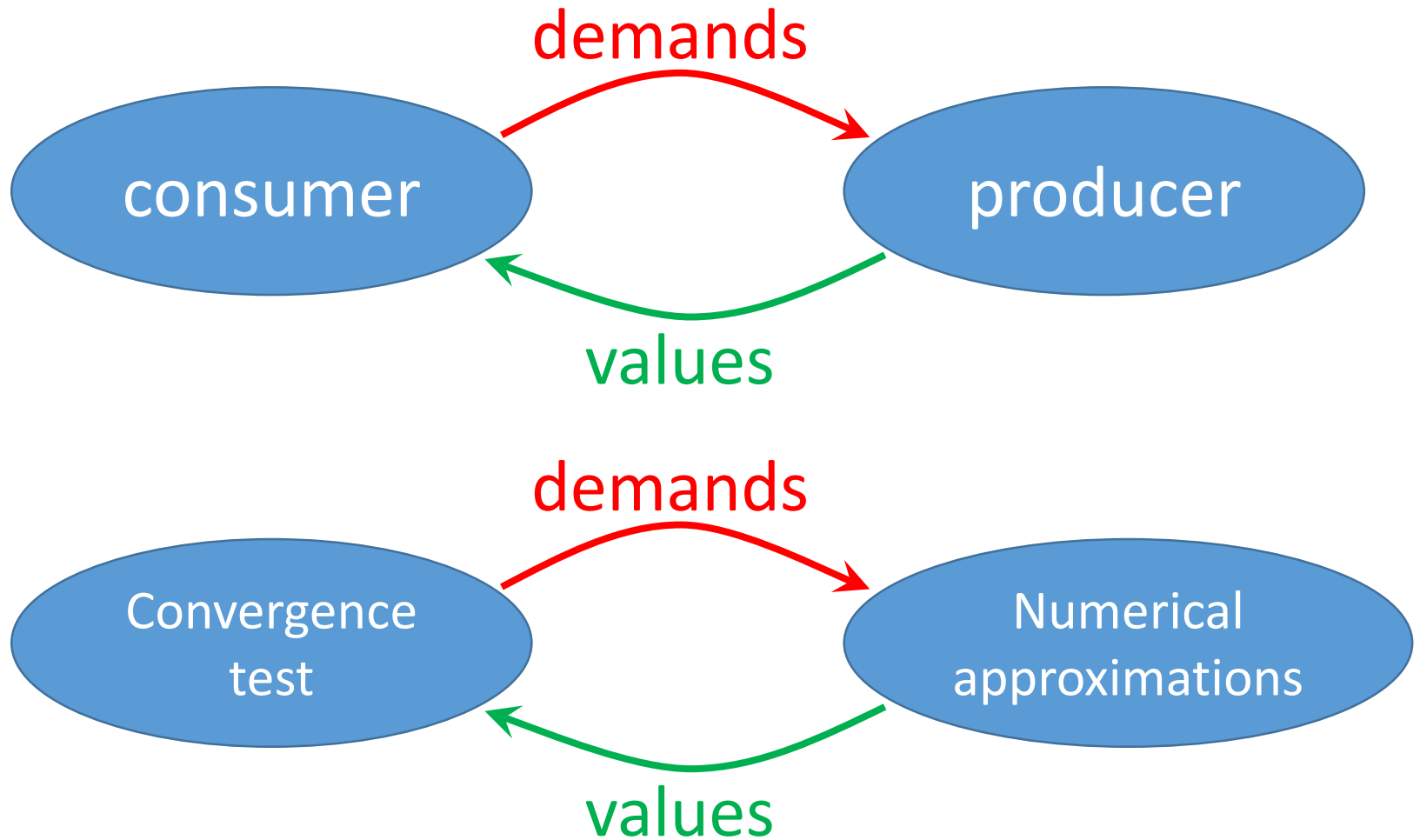
# Why Functional Programming Matters

John Hughes  
The University, Glasgow

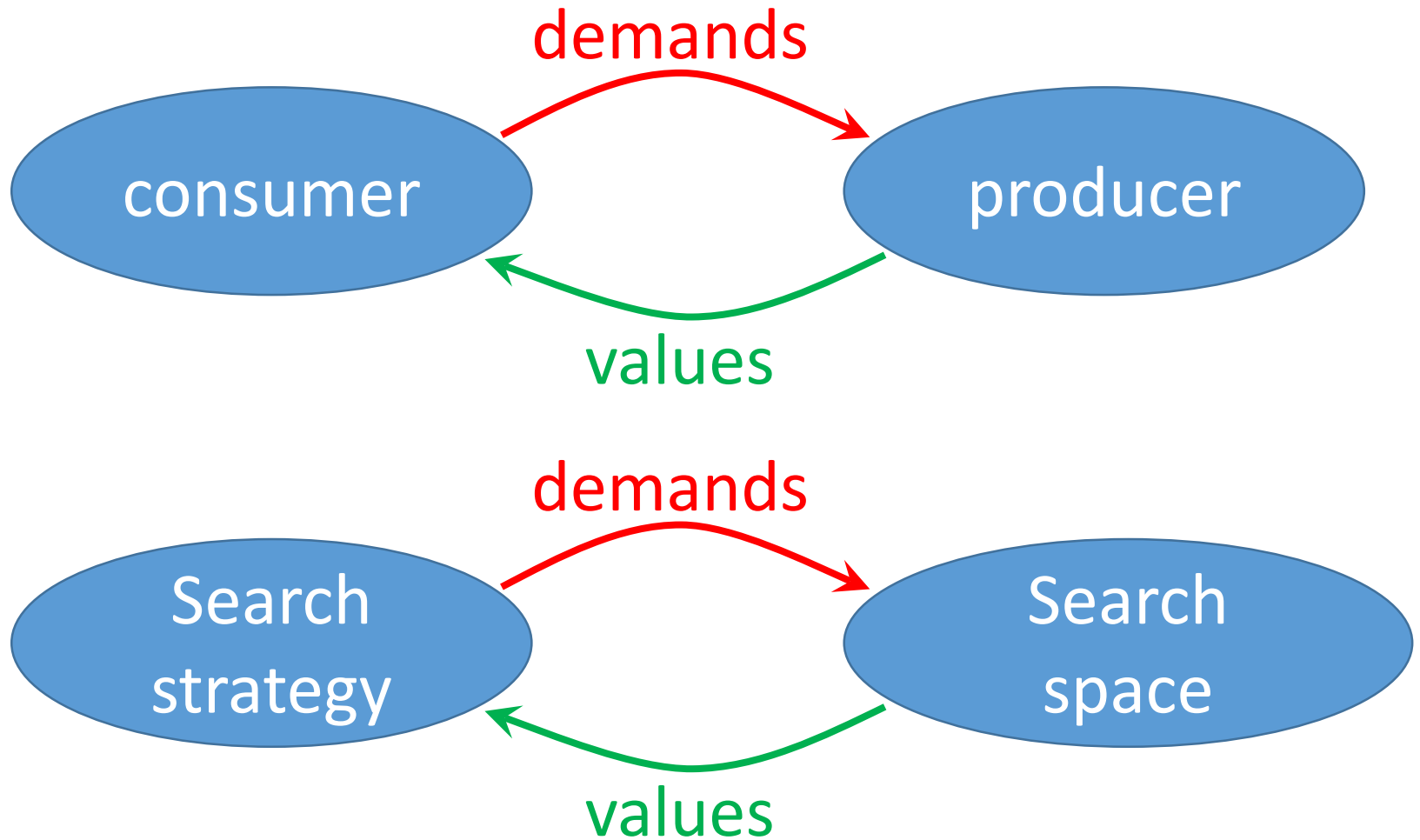


1990

# Lazy producer-consumer



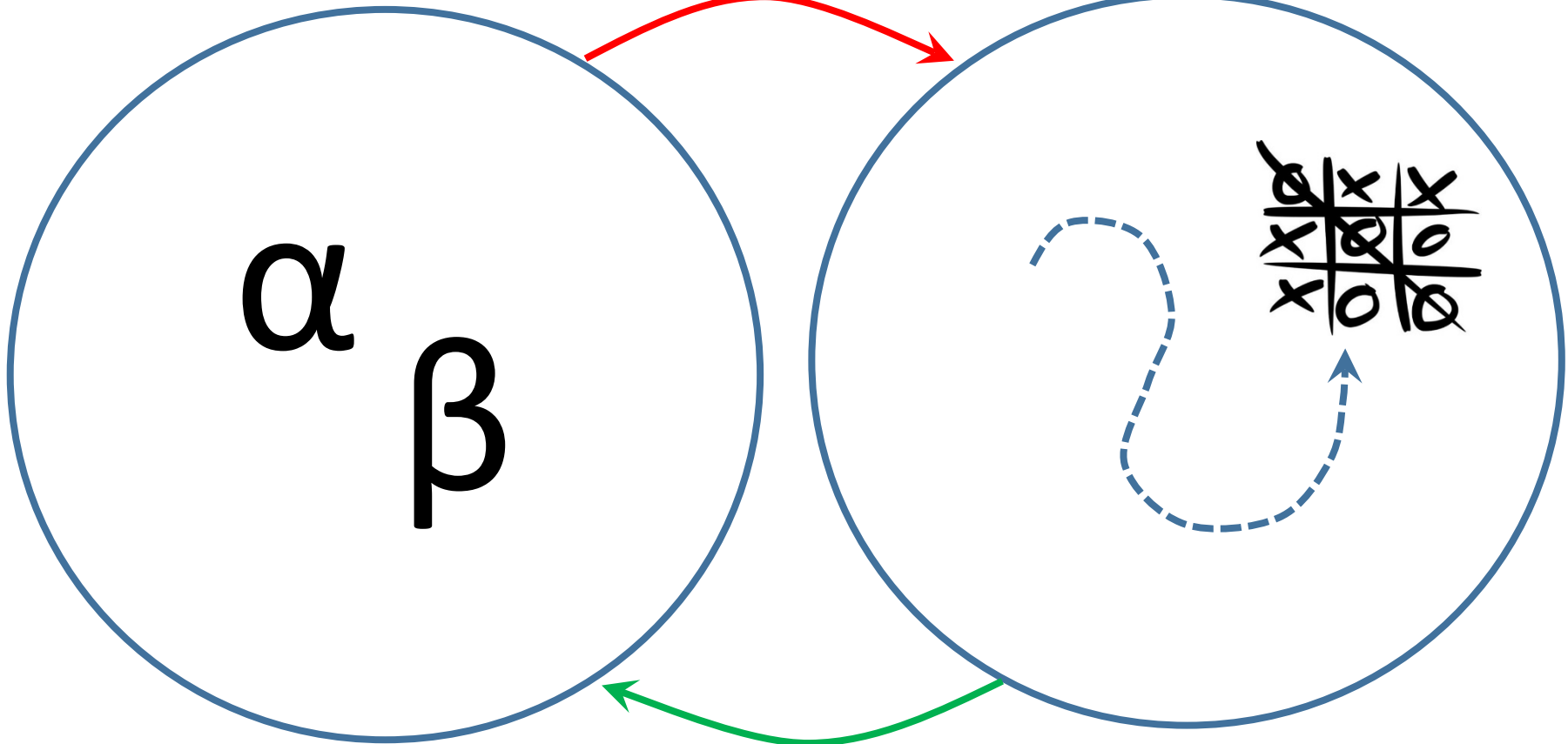
# Lazy producer-consumer



# Why Functional Programming Matters

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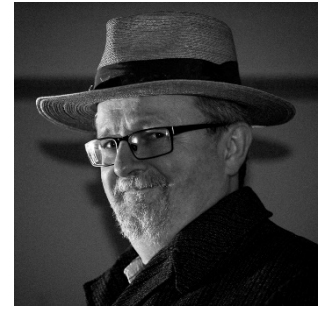
demands



values



# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



Koen Claessen  
Chalmers University of Technology  
koen@cs.chalmers.se

John Hughes  
Chalmers University of Technology  
rjmh@cs.chalmers.se

```
prop_reverse () ->
  ?FORALL (Xs, list (int ()),
    reverse (reverse (Xs)) == Xs) .
```

```
3> eqc:quickcheck (qc:prop_reverse ()) .
```

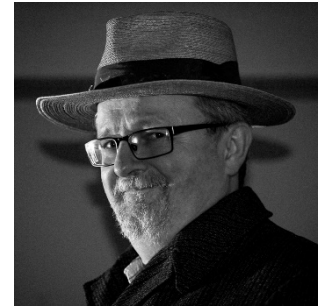
```
.....
.....
```

```
OK, passed 100 tests
```

```
true
```



# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



Koen Claessen  
Chalmers University of Technology  
koen@cs.chalmers.se

John Hughes  
Chalmers University of Technology  
rjmh@cs.chalmers.se

```
prop_wrong () ->  
  ?FORALL (Xs, list (int ()),  
          reverse (Xs) == Xs) .
```

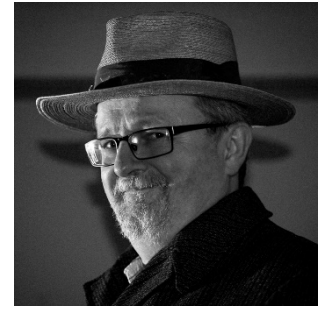
```
4> eqc:quickcheck (qc:prop_wrong ()) .  
Failed! After 1 tests.  
[-36,-29,20,31,-47,-63,80,-7,93,-87,-29,33,64,58]  
Shrinking xx.x.x..xx (4 times)
```

```
[0,1] ← minimal counterexample  
false
```



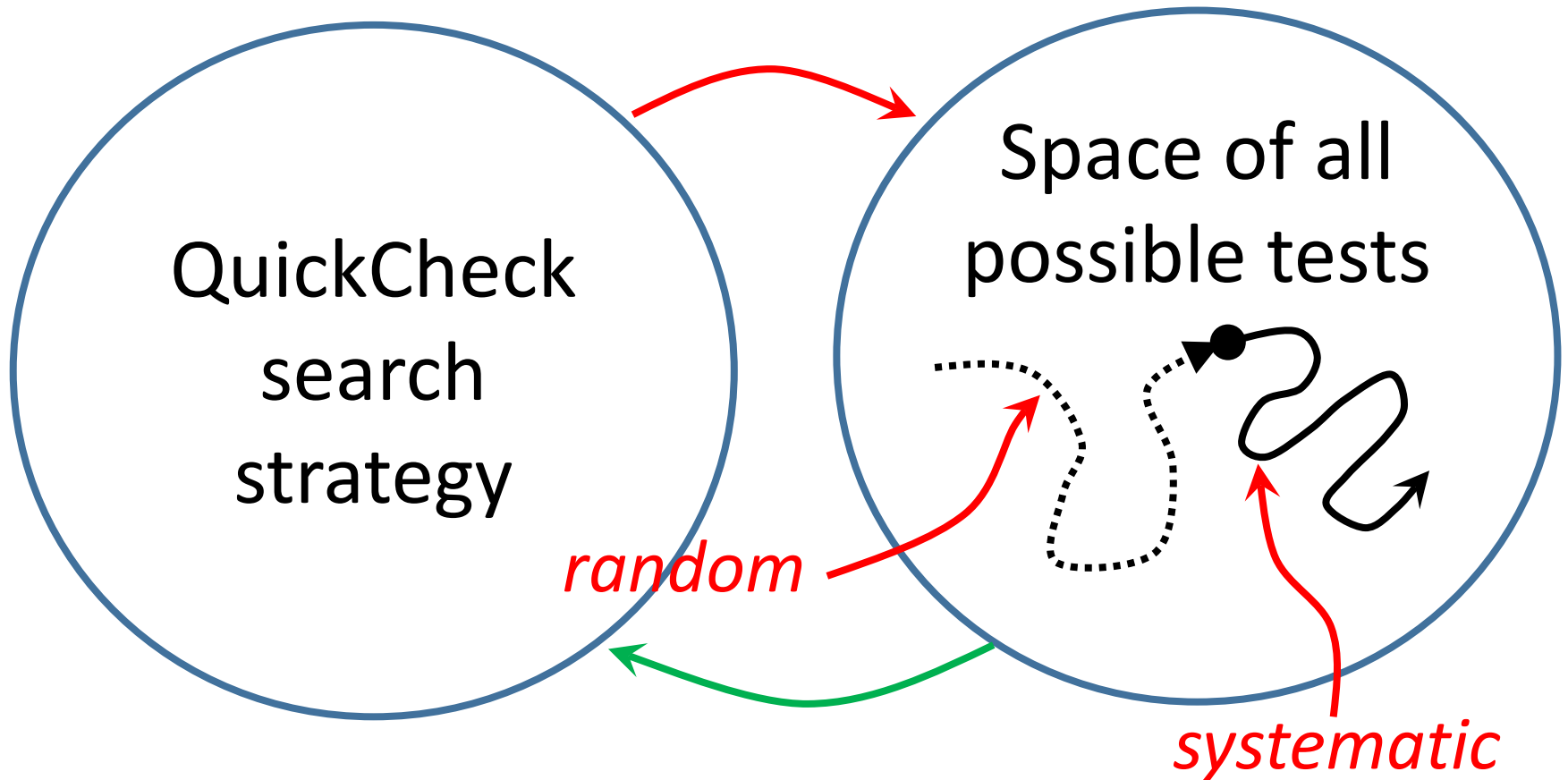


# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



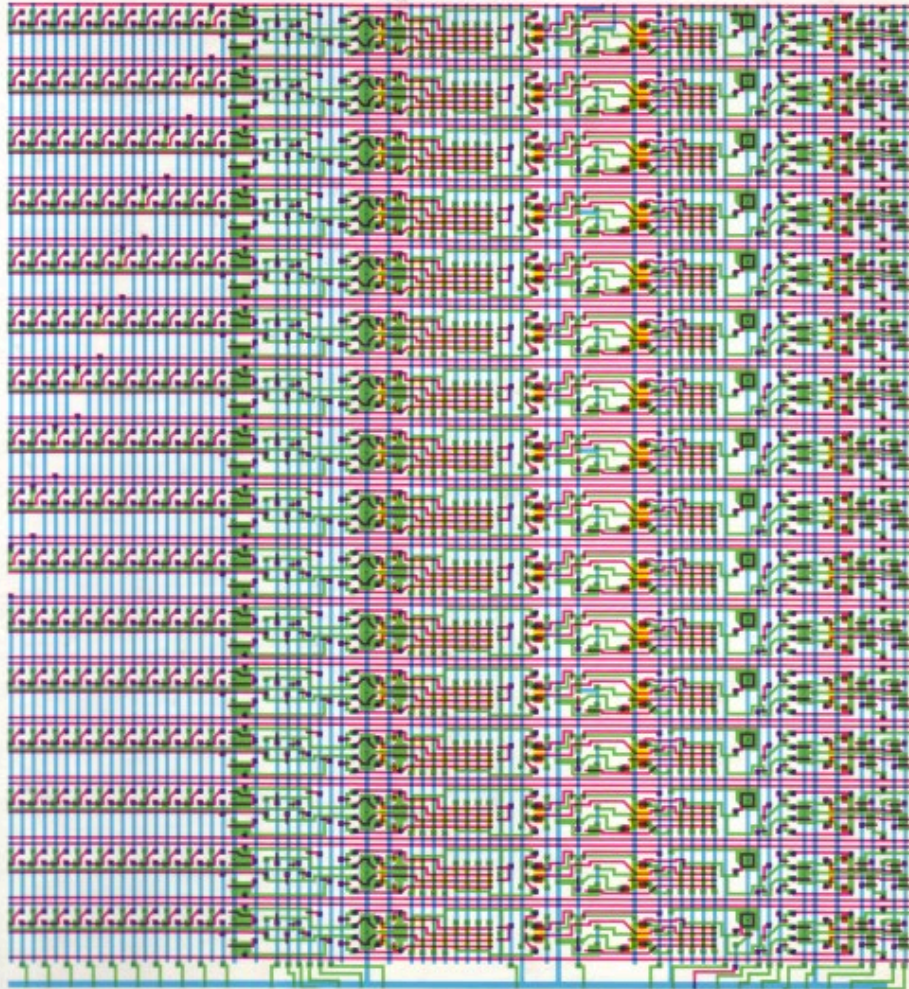
Koen Claessen  
Chalmers University of Technology  
koen@cs.chalmers.se

John Hughes  
Chalmers University of Technology  
rjmh@cs.chalmers.se




# INTRODUCTION TO **VLSI** SYSTEMS

CARVER MEAD • LYNN CONWAY

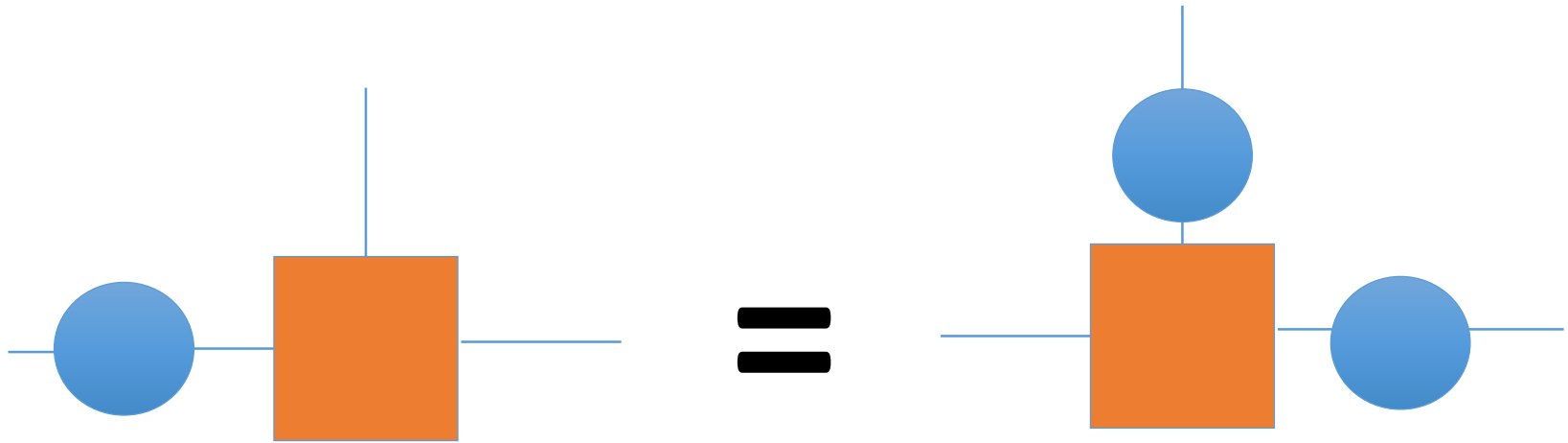


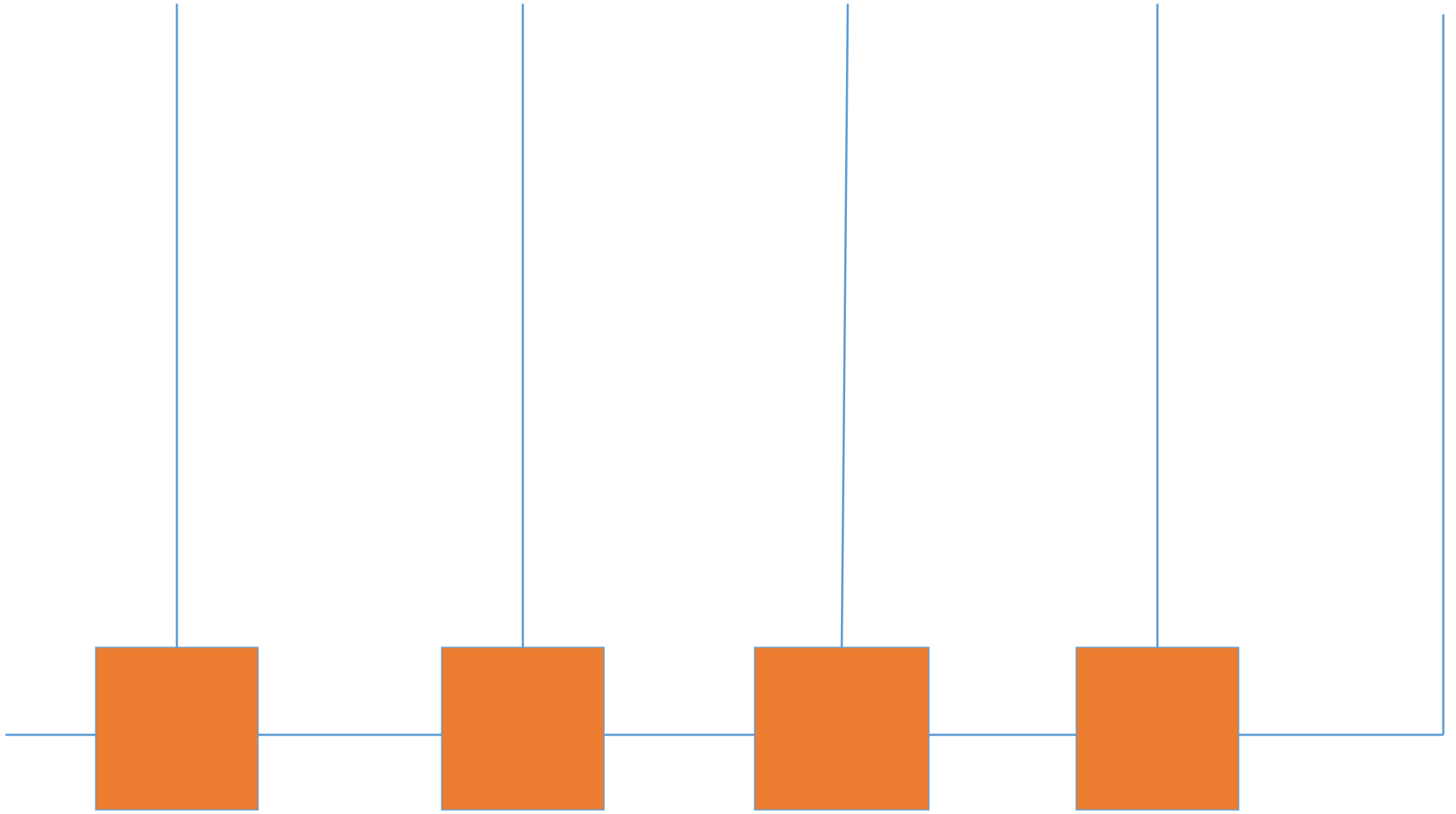


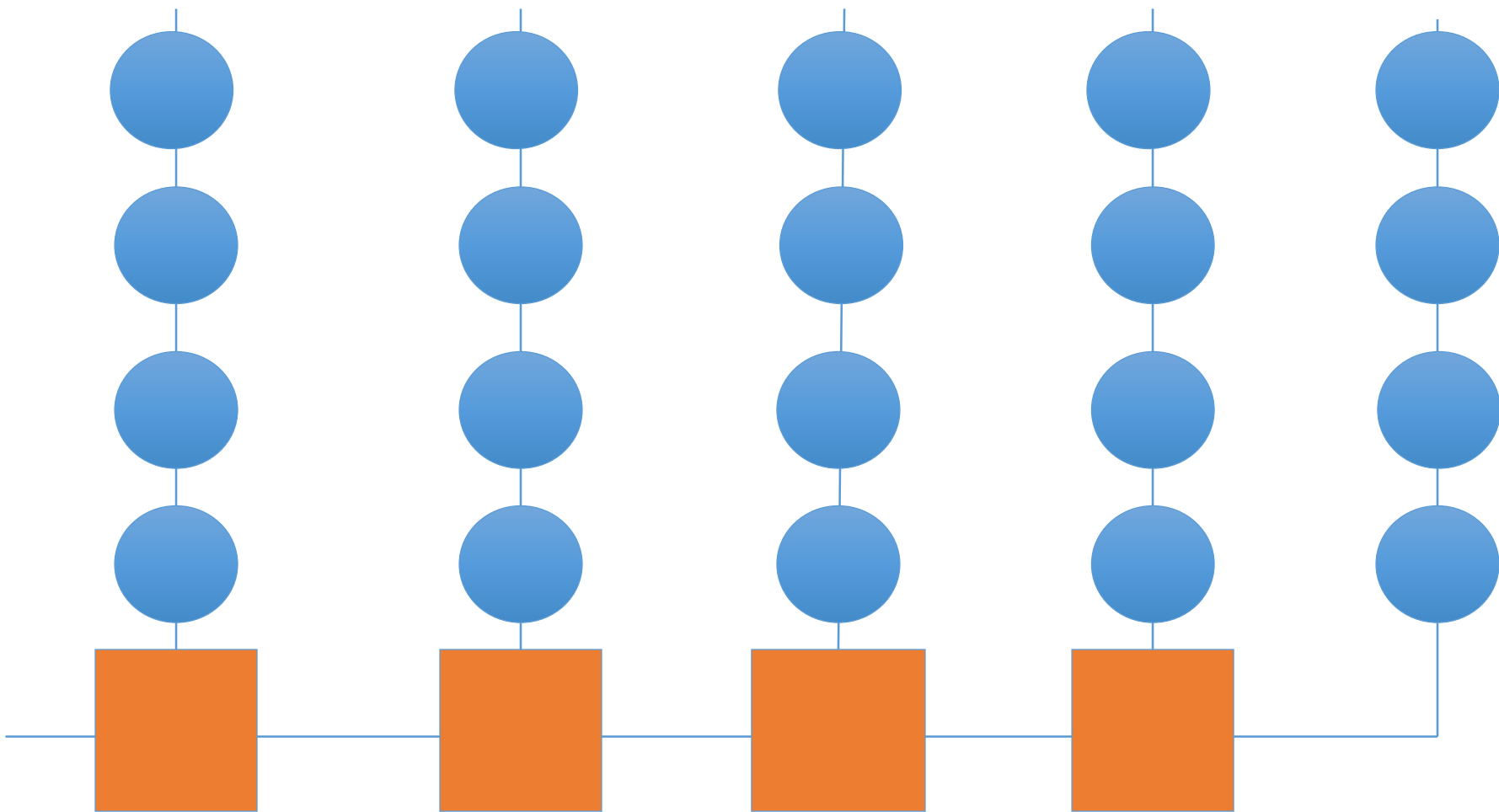
# muFP—Circuits as values

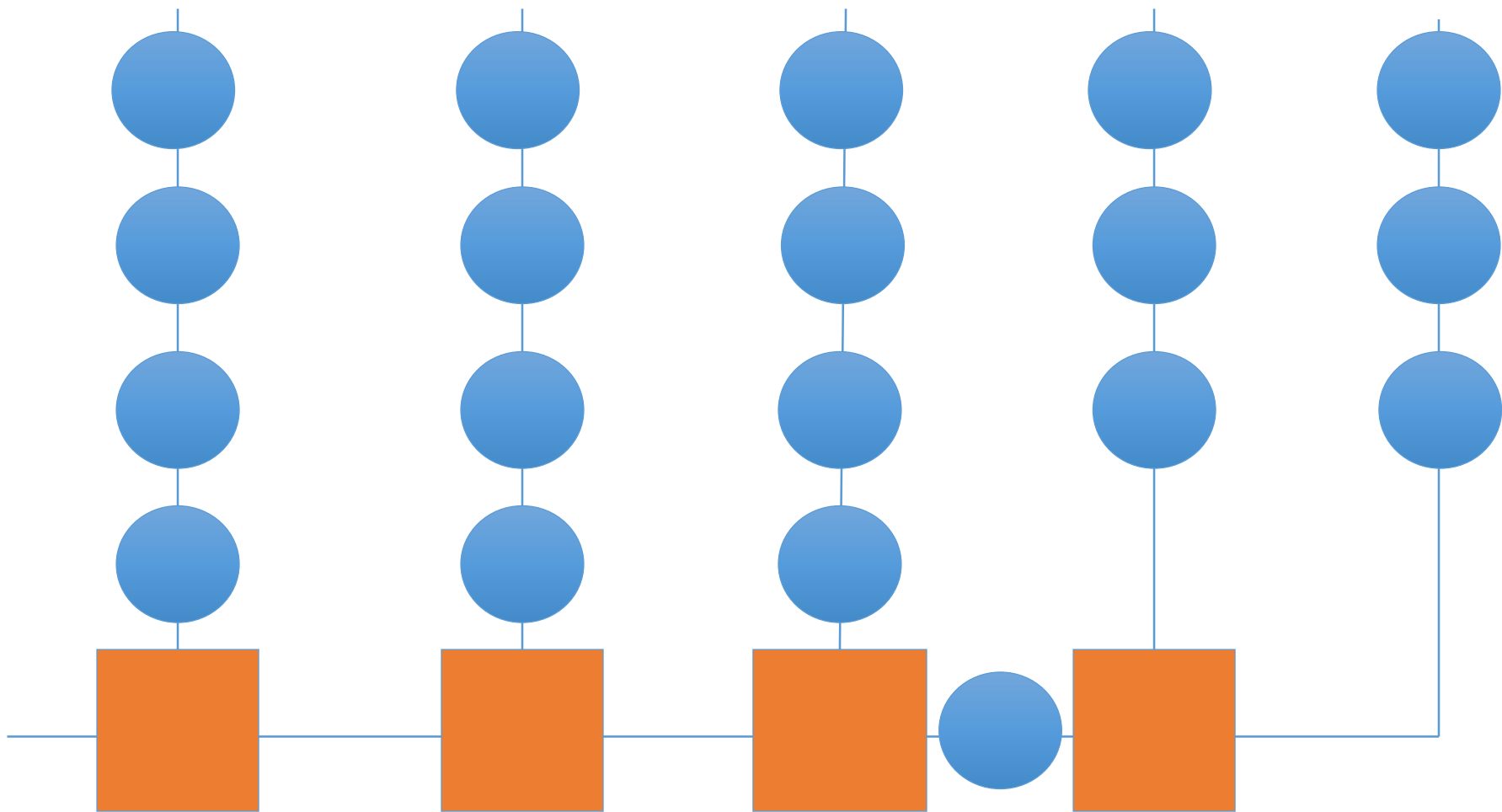
- Backus FP + one-clock-cycle delays 
- Inherits many combining forms and laws
- Good for reasoning about alternative designs

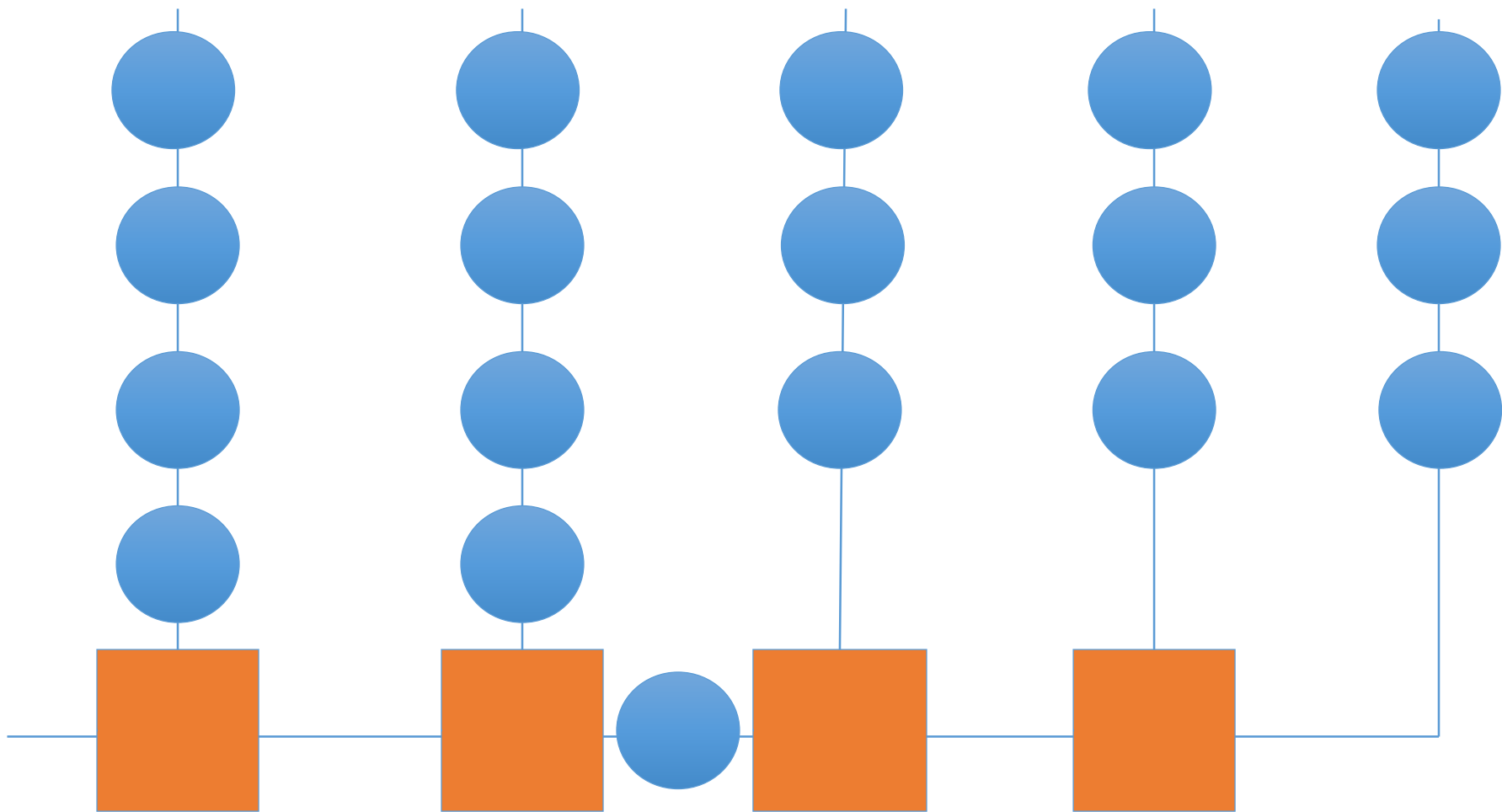
# Example law: “retiming”



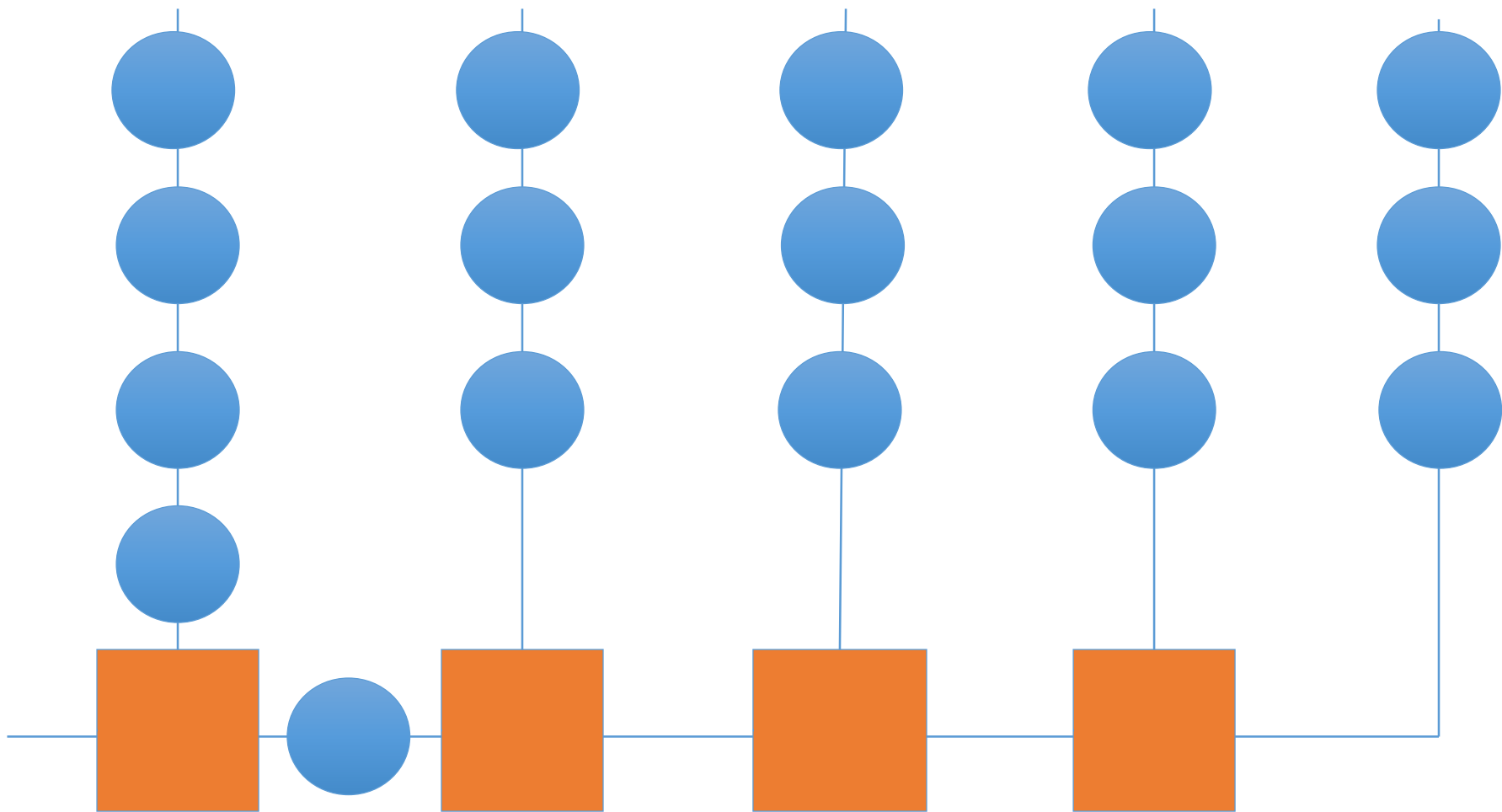


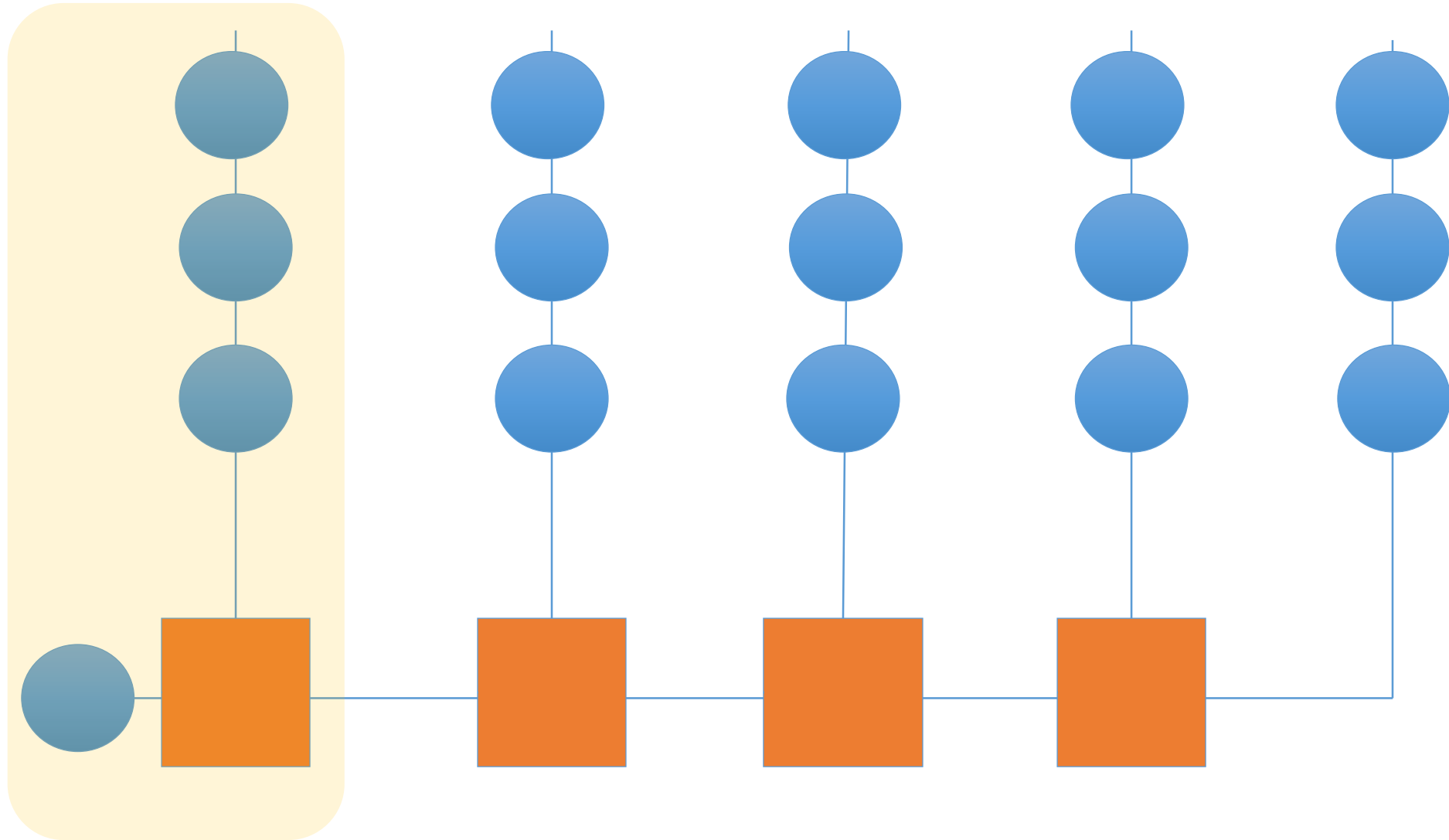


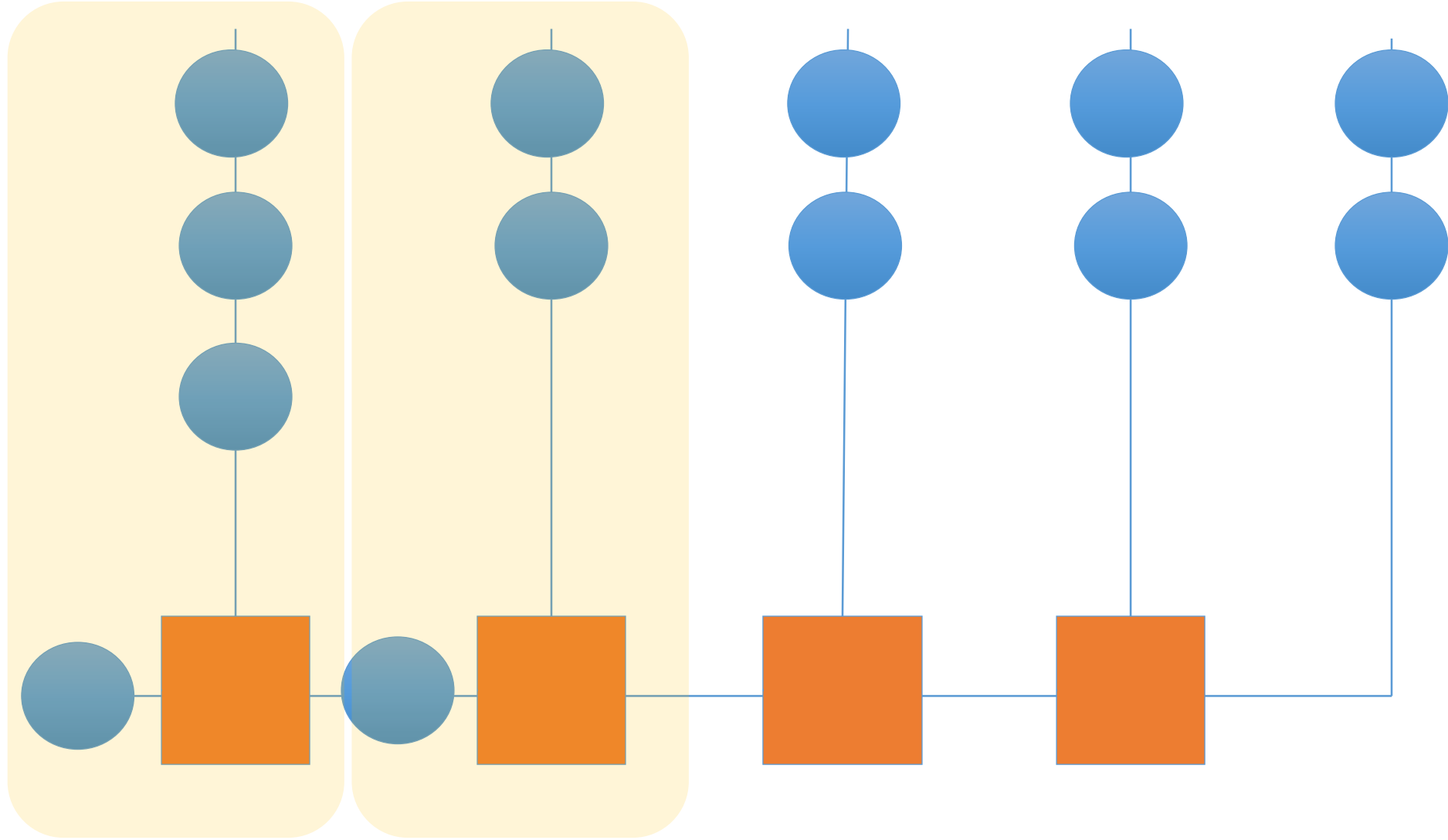


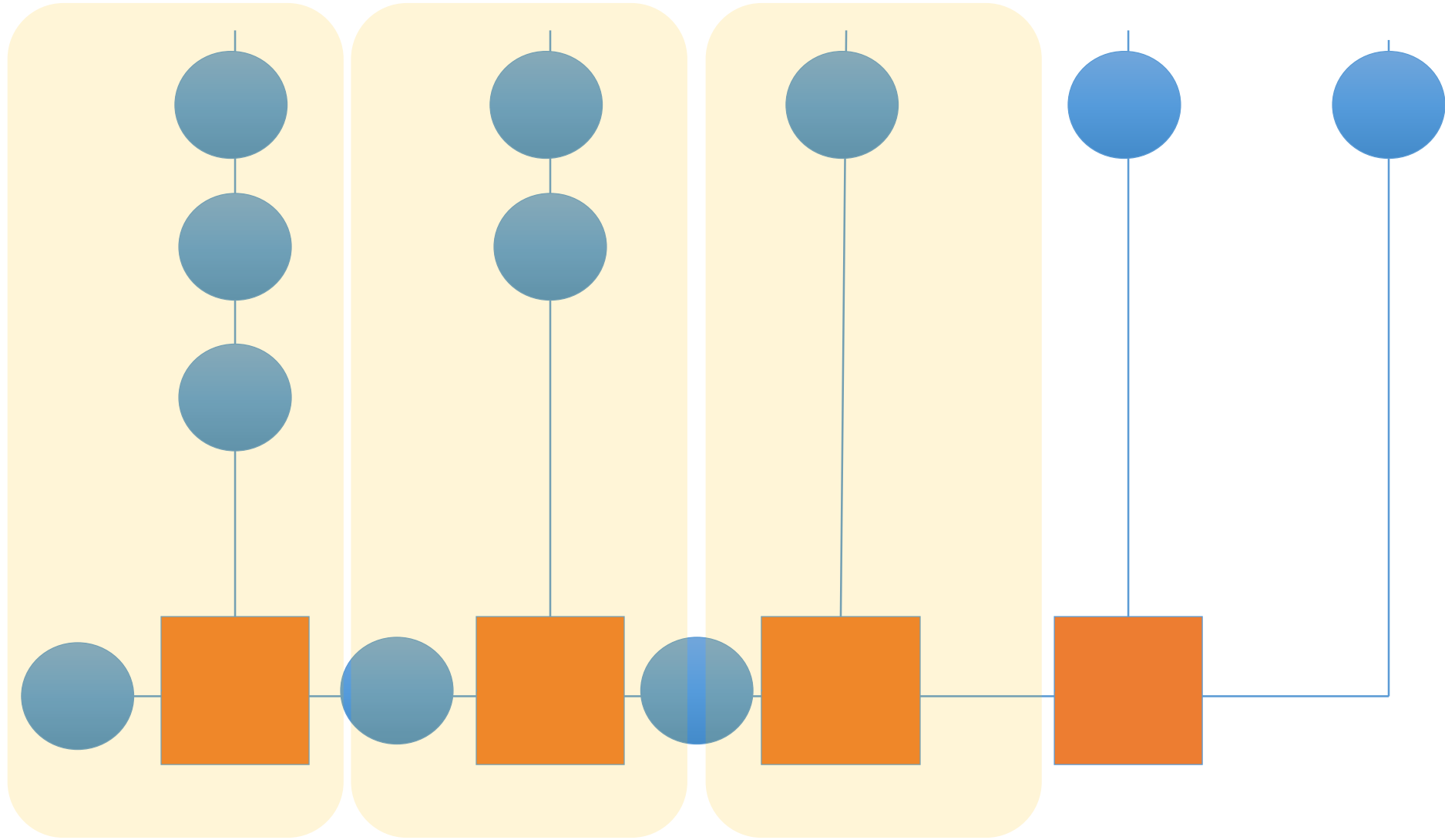


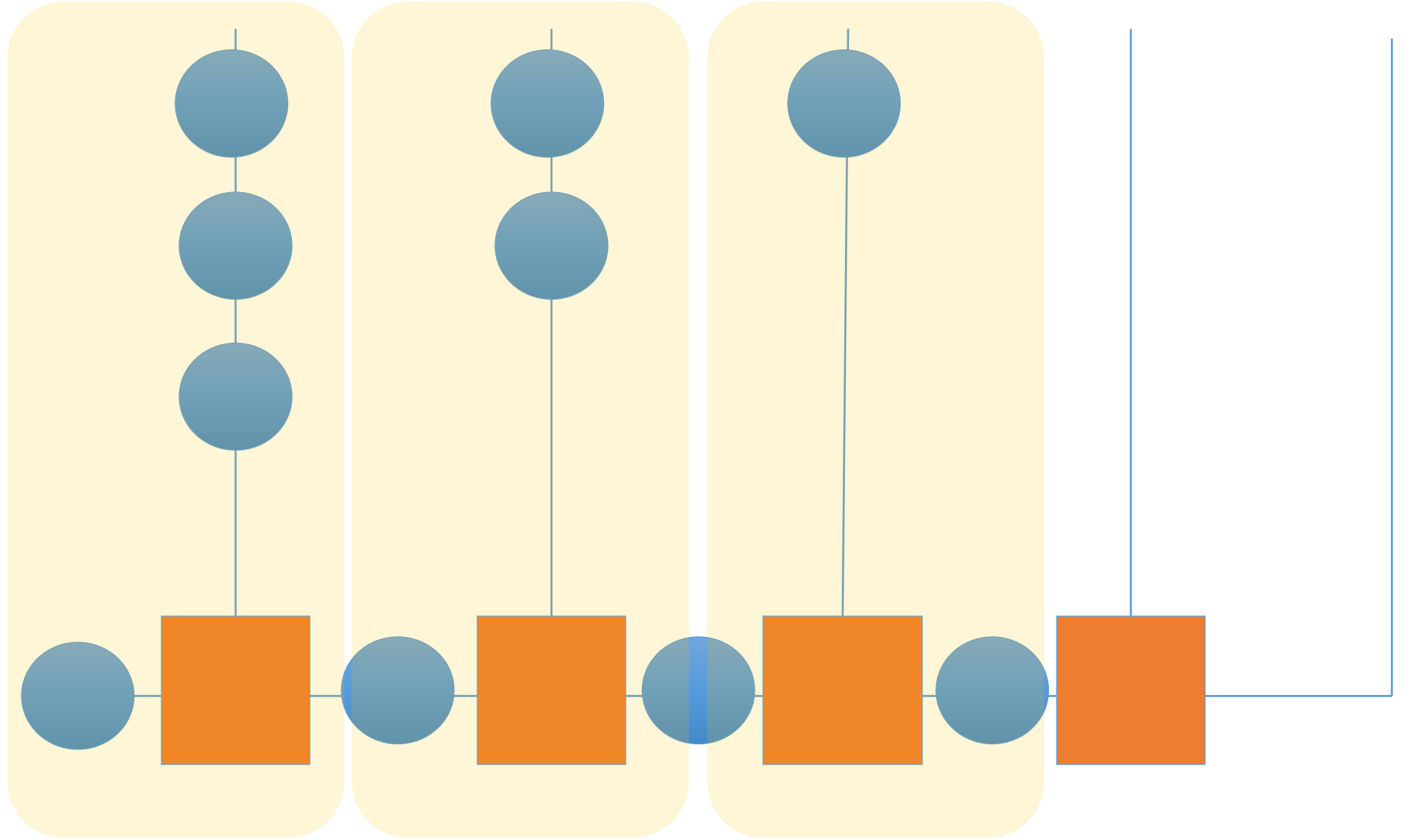












# Users!

## Plessey video motion estimation

“Using muFP, the array processing element was described in just one line of code and the complete array required four lines of muFP description. muFP enabled the effects of adding or moving data latches within the array to be assessed quickly.”

Bhandal et al, An array processor for video picture motion estimation, Systolic Array Processors, 1990, Prentice Hall

work with Plessey done by G. Jones and W. Luk

# Lava

muFP + Functional Geometry

Capture *semantics* of a circuit +  
relative *placement*

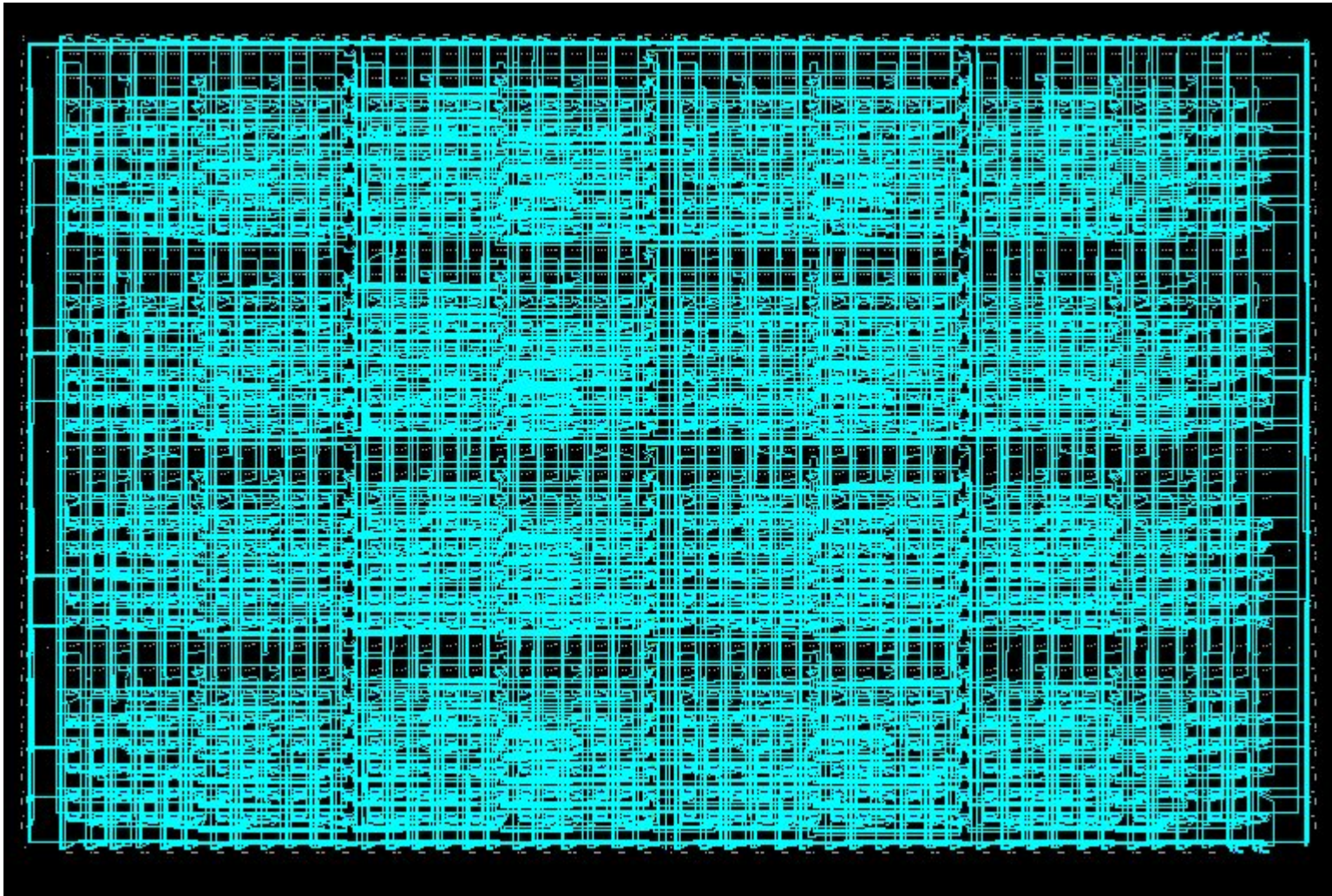
Programmer control of geometry!

➔ FPGA layouts on Xilinx chips



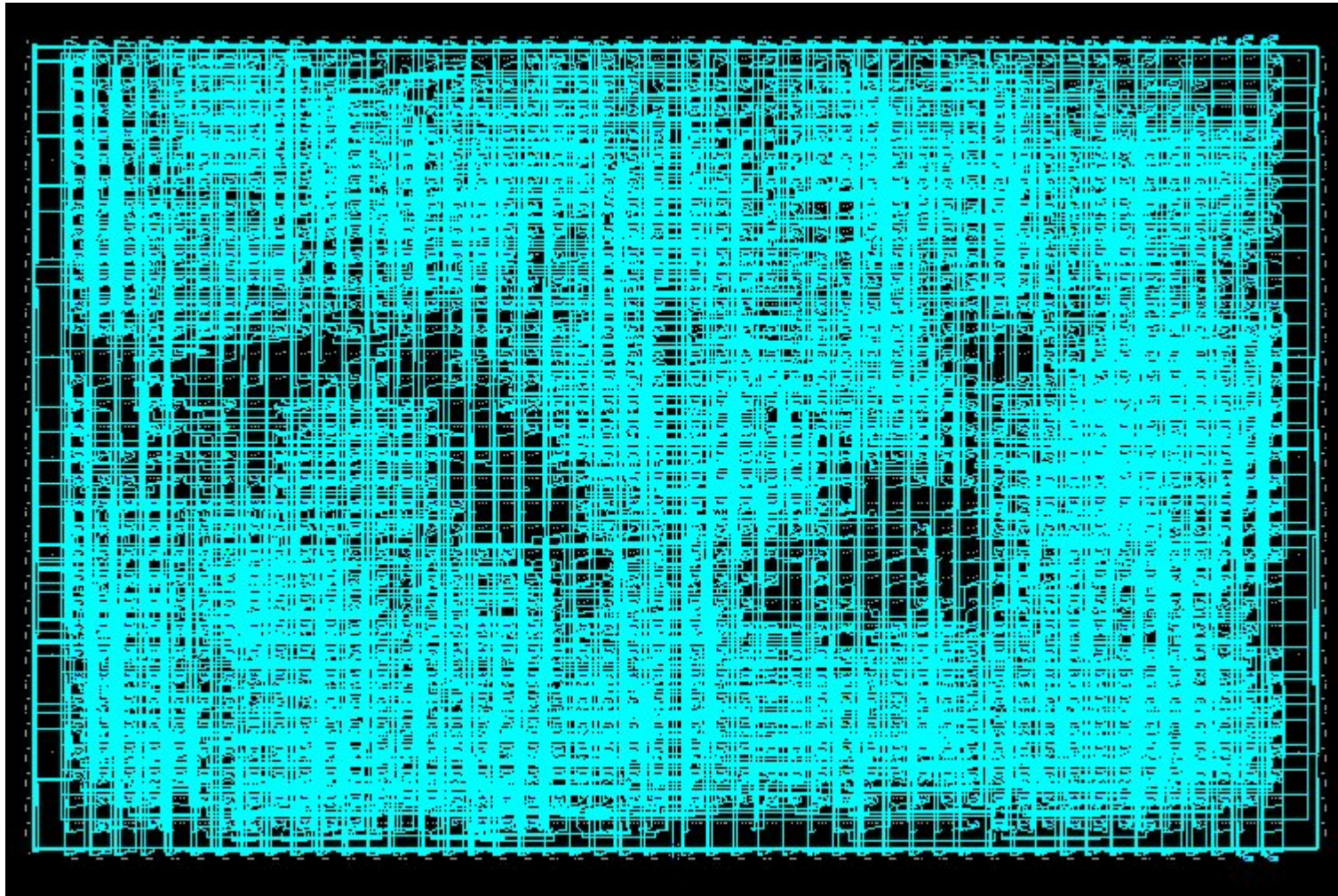
Satnam Singh  
at Xilinx

# Four adder trees from Lava





# From VHDL, without layout info



# Intel

$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 0$$

# Intel

$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 0$$

Flawed Pentium

$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 256$$

# \$475 million



Intel

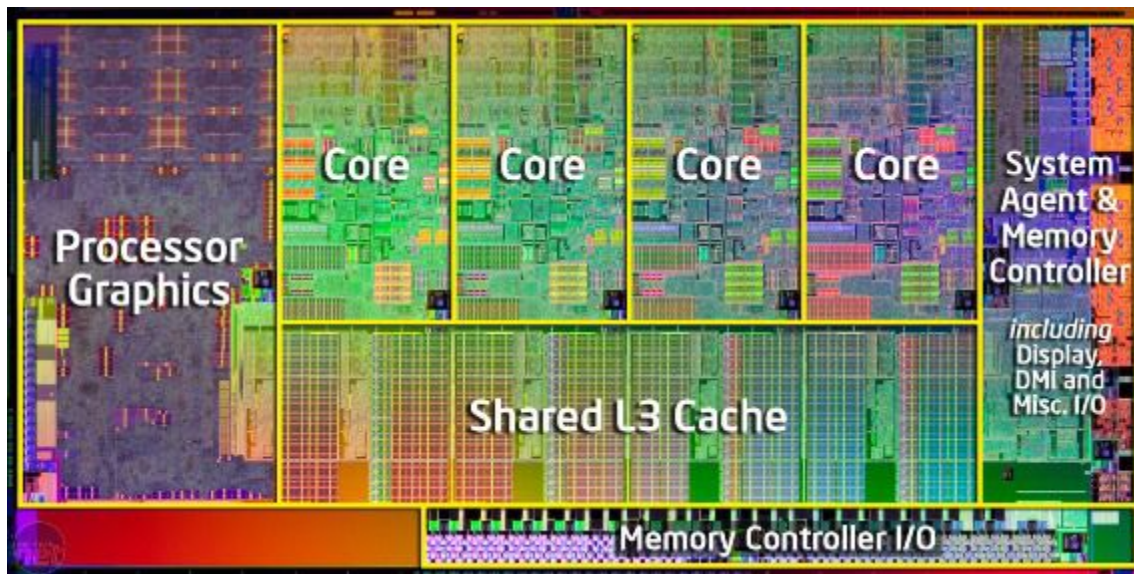


**fl** —lazy functional language

built-in decision procedures  
HW symbolic simulator

Forte System      1000s users

Thanks to Carl Seger (Intel)



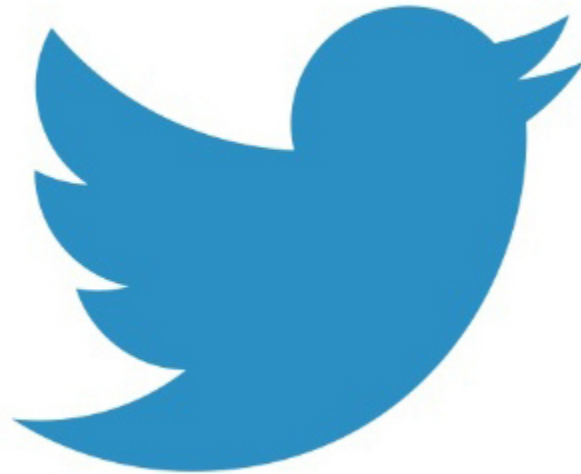


Google



The Scala logo consists of three horizontal, slightly curved red bars stacked vertically, with thin black gaps between them.

**Scala**







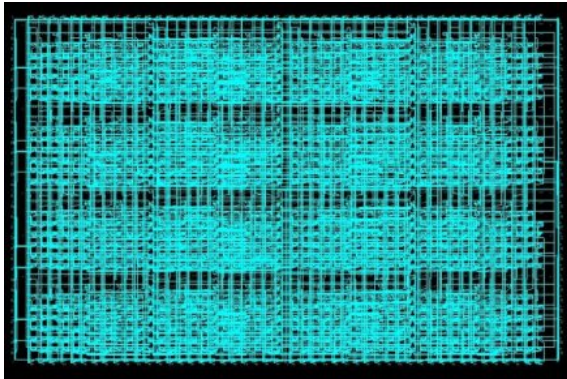
# FP courses

- TDA451 Functional Programming
  - Elective on IT line
- DAT280 Parallel functional programming
  - 3rd year+
  - Haskell, Erlang, emphasis on performance, guest lecturers from industry
- TDA342 Advanced functional programming
  - 3rd year+, MPALG
  - Key ideas behind functional libraries such as Akka

two  $f\ x = f\ (f\ x)$

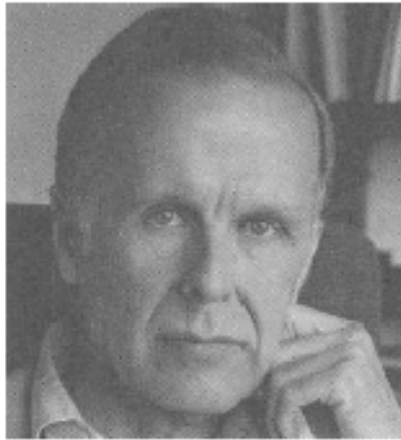
one  $f\ x = f\ x$

zero  $f\ x = x$



**Whole  
values**

**Combining  
forms**



**Simple  
laws**

**Functions as  
representations**